Methodology for Optimal Design of a Parking Lot

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(Reviewed by the Highway Division)

Abstract: A new approach to optimal design, maximum capacity, and the best layout for parking maneuvers, of a corner lot for parking spaces is introduced. Certain assumptions and practical design principles are used to derive the model, a system of nonlinear equations. The model is applied to a rectangular corner lot and solved by iteration using generated real data for combinations of compact and standard cars. The results show that in each case several layouts give the same maximum capacity. To select the maximum capacity corresponding to the best (safest) layout, an overall factor of easiness for parking maneuvers, called Ease, has been derived and used. The optimal solution of one case is than modified to fit other design specifications and plotted as an example of an actual parking layout for a corner lot. The model has several features: (1) Intuitively plausible principles of the initial design; (2) generality of equations to handle any car size, any combination of car sizes, and different angles in different regions; (3) it is easy to solve via applying data; and (4) it introduces Ease, an important factor to measure easiness to park, which gives the safest layout, etc. Further generalizations of the model are possible directions for future research.

Introduction

Parking is not a new phenomenon. An increasing population creates increased business and office accommodation in the city centers. This in turn brings increased traffic and increased parking requirements. Lack of proper parking facilities results in decentralization of business, which could cause economic problems. Most land-use projects, factories, professional buildings, shopping centers, etc., would be nearly valueless if people could not find nearby places for parking their vehicles. Parking facilities are an essential part of today's economy.

The most important aspect of a parking facility is its design, and the most important consideration in the design is the capacity. One should first design for maximum capacity, then modify the design to fit other design specifications.

In this paper the design of one type of off-street parking facility, a corner lot surrounded by two streets intersecting at the corner, is discussed. The optimal capacity is to be found. By this we mean the maximum capacity along with the most comfortable (easiest) layout for parking maneuvers.

Certain assumptions and three intuitive design principles lead to the initial layout, design 3. This, upon considering traffic flow and assigning different parking angles α_i, $i = 1, \ldots, 6$ to different regions, gives design 4. The most complete initial layout, design 5, is then reached by imposing symmetry, introducing rectangles close to the corners, and renumbering the

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TABLE 1. Selection of Parking Lot Design Parameters in Feet from *Parking Design Manual* (n.d.) and Ramsey and Sleeper (1981)

Parking angle α (1)	Stall width SSW (2)	Curb length $SCL(\alpha)$ (3)	Stall depth $SSD(\alpha)$ (4)	Aisle width $SAW(\alpha)$ (5)
(*a*) Compact Car[a]				
0	7.5	21.50	7.50	11
20	7.5	22.17	13.08	10
30	7.5	15.00	15.33	10
40	7.5	11.75	17.08	11
45	7.5	10.58	17.75	12.5
50	7.5	9.75	18.33	11.5
60	7.5	8.75	19.00	17.5
70	7.5	8.00	19.08	18.5
80	7.5	7.67	18.67	23
90	7.5	7.50	17.58	24
(*b*) Standard Car[b]				
0	8.5	23.00	8.50	12
20	8.5	24.92	14.50	11
30	8.5	17.00	16.92	11
40	8.5	13.25	18.75	12
45	8.5	12.00	19.42	13.5
50	8.5	11.17	20.00	12.5
60	8.5	9.83	20.75	18.5
70	8.5	9.00	20.83	19.5
80	8.5	8.67	20.25	24
90	8.5	8.50	19.00	24

[a] $SITR = 11.833$ and $SOTR = 21.500$.
[b] $ITR = 12.600$ and $OTR = 22.417$.

different regions. Symmetry is used to reduce parking confusion, and rectangles (regions 4 and 5) are to be used for small cars and straight-in parking. Next, the model, a system of nonlinear equations (Eqs. 2–10) is derived subject to practical considerations and a width constraint (Eq. 11). Table 1 represents our selection of parameter design data from the *Parking Design Manual* (n.d.) and Ramsey and Sleeper (1981) for compact and standard cars. Since such data are available for only a few parking angles, we extend them for all angles ranging from 0°–90°, using a natural cubic spline algorithm. Using the data, the model is then solved by iteration and the results, all the design parameters, are generated for all combinations of compact and standard cars in eight cases. Sample cases are shown in Tables 2–5. Since in each table several designs give the same maximum number of spaces, an overall comfort factor, denoted Ease, shown in the last column, is calculated by $\bar{C}$ in Eq. 12 and used as a guideline to highlight the optimal design of all cases. Table 6 shows eight optimal designs, each corresponding to the highlighted solution of one case for a combination of compact and standard cars. As an example, out of the eight optimal solutions for different cases, the one highlighted in Table 2 is applied to design 5 to give the final design 6. We note this final design uses assumptions 5 and 6 to have one driveway for both entrance and exit by taking off the three spaces in the upper region 4 and one additional adjacent space from the upper region 1. Further, we have designed an automatic gate at the entrance and a booth by the exit on

TABLE 2. Sample Table, Designing for Standard Cars in Regions 1, 2, 3, and Compact Cars in 4, 5

α_1 (1)	α_2 (2)	α_3 (3)	N_1 (4)	N_2 (5)	N_3 (6)	N_4 (7)	N_5 (8)	W_1 (9)	W_2 (10)	ΣN_i (11)	Ease (12)
50	45	80	24	14	12	6	2	13.59	24.00	58	3.33
50	45	85	24	14	12	6	2	13.59	24.68	58	3.30
50	45	90	24	14	12	6	2	13.59	24.00	58	3.27
50	50	0	28	22	4	4	2	12.73	19.75	58	2.86
50	50	45	26	18	8	4	2	12.73	19.75	58	3.10
50	50	55	24	18	10	4	2	12.73	19.75	58	2.95
50	50	60	24	18	10	4	2	12.73	19.75	58	2.89
50	50	65	24	18	10	4	2	12.73	19.75	58	2.84
50	50	70	24	18	10	4	2	12.73	19.75	58	2.79
50	50	80	24	16	12	6	2	13.59	24.00	60	3.08
50	50	85	24	16	12	6	2	12.73	24.68	60	3.06
50	50	90	24	16	12	6	2	12.73	24.00	60	3.02

TABLE 3. Sample Table, Designing for Standard Cars in Regions 1, 2 and Compact Cars in 3, 4, 5

α_1 (1)	α_2 (2)	α_3 (3)	N_1 (4)	N_2 (5)	N_3 (6)	N_4 (7)	N_5 (8)	W_1 (9)	W_2 (10)	ΣN_i (11)	Ease (12)
0	75	65	12	26	16	4	4	21.97	18.33	58	2.42
0	75	70	12	26	16	4	4	21.97	18.50	58	2.36
0	75	90	12	24	18	6	4	21.97	24.00	58	2.75
40	45	90	20	16	14	6	2	14.84	24.00	58	2.86
40	50	75	20	18	14	4	2	13.98	20.53	58	2.28
40	50	80	20	16	14	6	2	13.98	23.00	58	2.69
40	50	85	20	16	14	6	2	13.98	24.06	58	2.67
40	50	90	20	16	14	6	2	13.98	24.00	58	2.64
45	40	55	24	16	10	4	4	15.09	15.97	58	3.43
45	40	80	22	14	12	6	4	15.09	23.00	58	3.67
45	40	90	22	14	12	6	4	15.09	24.00	58	3.63
45	45	45	24	18	10	4	2	14.17	17.11	58	2.85
45	45	50	24	18	10	4	2	14.17	17.11	58	2.74
45	45	60	24	16	12	4	2	14.17	17.50	58	2.67
45	45	65	24	16	12	4	2	14.17	18.33	58	2.63
45	45	70	24	16	12	4	2	14.17	18.50	58	2.59
45	45	75	24	16	12	4	2	14.17	20.53	58	2.58
45	45	85	22	14	14	6	2	14.17	24.06	58	2.94
45	45	90	22	16	14	6	2	14.17	24.00	60	2.81
50	40	90	24	14	14	6	2	14.51	24.00	60	3.20
50	45	60	26	16	12	4	2	13.59	18.01	60	2.63
50	45	65	26	16	12	4	2	13.59	18.33	60	2.58
50	45	70	26	16	12	4	2	13.59	18.50	60	2.54
50	45	75	26	16	12	4	2	13.59	20.53	60	2.53
50	45	90	24	16	14	6	2	13.59	24.00	62	2.75
50	50	60	26	18	12	4	2	12.73	19.75	62	2.33
50	50	65	26	18	12	4	2	12.73	19.75	62	2.27
50	50	70	26	18	12	4	2	12.73	19.75	62	2.22
50	50	75	26	18	12	4	2	12.73	20.53	62	2.19
50	50	90	24	16	14	6	2	12.73	24.00	62	2.52

TABLE 4. Sample Table, Designing for Standard Cars in Regions 2 and Compact Cars in 1, 3, 4, 5

α_1 (1)	α_2 (2)	α_3 (3)	N_1 (4)	N_2 (5)	N_3 (6)	N_4 (7)	N_5 (8)	W_1 (9)	W_2 (10)	ΣN_i (11)	Ease (12)
45	40	80	26	14	12	6	4	16.76	23.00	62	4.43
45	40	90	26	14	12	6	4	16.76	24.00	62	4.38
45	45	45	28	18	8	4	4	15.84	15.21	62	4.11
45	45	50	28	18	10	4	4	15.84	15.21	64	3.99
45	45	90	26	16	12	6	4	15.84	24.00	64	4.04
45	50	45	28	20	10	4	2	14.98	16.09	64	3.52
45	50	50	28	20	10	4	2	14.98	16.09	64	3.43
45	50	55	26	20	12	4	2	14.98	16.09	64	3.34
45	50	70	26	18	14	4	2	14.98	18.50	64	3.21
45	50	75	26	18	14	4	2	14.98	20.53	64	3.20
45	50	80	26	16	14	6	2	14.98	23.00	64	3.51
45	50	85	26	16	14	6	2	14.98	24.06	64	3.49
45	50	90	26	16	14	6	2	14.98	24.00	64	3.46
50	40	70	30	14	12	4	4	16.17	18.50	64	3.75
50	40	75	30	14	12	4	4	16.17	20.53	64	3.75
50	40	80	28	14	12	6	4	16.17	23.00	64	4.05
50	40	90	28	14	12	6	4	16.17	24.00	64	4.00
50	45	45	30	18	8	4	4	15.26	15.79	64	3.74
50	45	50	30	18	8	4	4	15.26	15.79	64	3.67
50	45	55	30	18	10	4	4	15.26	15.79	66	3.56
50	45	70	30	16	12	4	4	15.26	18.50	66	3.45
50	45	75	30	16	12	4	4	15.26	20.53	66	3.44
50	45	90	28	16	12	6	4	15.26	24.00	66	3.69
50	50	45	30	20	10	4	2	14.40	16.80	66	3.20
50	50	50	30	20	10	4	2	14.40	16.80	66	3.11
50	50	55	30	18	12	4	2	14.40	16.80	66	3.04
50	50	60	30	18	12	4	2	14.40	17.50	66	2.99
50	50	65	30	18	12	4	2	14.40	18.33	66	2.96
50	50	70	30	18	12	4	2	14.40	18.50	66	2.92
50	50	75	30	18	14	4	2	14.40	20.53	68	2.87
55	45	75	32	16	14	4	2	14.84	20.53	68	2.85
55	45	90	30	16	14	6	2	14.84	24.00	68	3.11
55	50	45	32	20	10	4	2	13.98	17.39	68	2.96
55	50	75	32	18	14	4	2	13.98	20.53	70	2.62

the available adjacent space (upper corner B) for parking payments. All the available spaces near the corners, lower B, lower A, upper A, and lower and upper C, are to be used for storage, motorcycle, and bicycle parking. Other specifications, such as lighting, snow removal, landscaping etc., can easily be incorporated as needed. A listing of the PASCAL source code that produces the results is available from the writers.

ASSUMPTIONS

Based upon the references on the topic, for simplifications and practical purposes the following assumptions have been made:

1. The layout complies with all local, state, and federal parking codes.
2. The lot surface is assumed to be flat and rectangular, with a fixed width, W = 100 ft but a varying length L (in the computations we have selected L = 200 ft).

TABLE 5. Sample Table, Designing for Standard Cars in Regions 3 and Compact Cars in 1, 2, 4, 5

α_1 (1)	α_2 (2)	α_3 (3)	N_1 (4)	N_2 (5)	N_3 (6)	N_4 (7)	N_5 (8)	W_1 (9)	W_2 (10)	ΣN_i (11)	Ease (12)
0	80	55	14	32	10	2	6	24.48	14.96	62	2.86
0	85	35	14	36	6	2	6	24.64	11.83	62	2.92
0	85	45	14	34	8	2	6	24.64	13.50	62	2.85
0	85	50	14	34	8	2	6	24.64	12.50	62	2.77
0	85	55	14	32	10	2	6	24.64	14.96	62	2.78
0	90	0	16	42	4	2	6	24.92	12.00	62	2.73
0	90	35	14	36	6	2	6	24.92	11.83	62	2.85
0	90	40	14	36	6	2	6	24.92	12.00	62	2.77
0	90	45	14	34	8	2	6	24.92	13.50	62	2.78
0	90	50	14	36	8	2	6	24.92	12.50	64	2.65
0	90	55	14	34	10	2	6	24.92	14.96	64	2.67
45	55	0	30	28	4	4	4	15.65	15.39	66	3.71
50	55	0	32	26	4	4	4	15.07	16.00	66	3.38
50	55	40	30	22	6	4	4	15.07	16.00	66	3.47
50	55	50	28	22	8	4	4	15.07	16.00	66	3.34
50	55	55	28	22	8	4	4	15.07	16.00	66	3.29
50	55	65	28	20	10	4	4	15.07	19.31	66	3.26
50	55	70	28	20	10	4	4	15.07	19.50	66	3.23
50	55	75	28	20	10	4	4	15.07	21.59	66	3.22
50	55	80	28	18	10	6	4	15.07	24.00	66	3.53
50	55	85	28	18	10	6	4	15.07	24.68	66	3.51
50	55	90	28	18	10	6	4	15.07	24.00	66	3.48
55	50	0	34	24	4	4	4	15.33	15.72	66	3.36
55	50	40	32	20	6	4	4	15.33	15.72	66	3.43
55	50	55	30	20	8	4	4	15.33	15.72	66	3.32
55	50	50	30	20	8	4	4	15.33	15.72	66	3.27
55	50	65	30	18	10	4	4	15.33	19.31	66	3.23
55	50	70	30	18	10	4	4	15.33	19.50	66	3.19
55	50	75	30	18	10	4	4	15.33	21.59	66	3.19
55	50	85	30	16	10	6	4	15.33	24.68	66	3.46
55	50	90	30	16	10	6	4	15.33	24.00	66	3.44
55	55	0	34	26	4	4	2	14.65	16.49	68	2.74
55	55	55	30	22	10	4	2	14.65	16.49	68	2.75
55	55	70	30	20	12	4	2	14.65	19.50	68	2.66
55	55	75	30	20	12	4	2	14.65	21.59	68	2.66
55	55	85	30	18	12	6	2	14.65	24.68	68	2.94
55	55	90	30	18	12	6	2	14.65	24.00	68	2.91
90	0	55	40	12	8	2	6	24.92	14.96	68	2.54

3. The layout is designed for one-way traffic only, no matter how wide the aisles are.
4. The lot is designed for head-in parking only.
5. The lot is small enough to be handled with one driveway of a minimum width of 30 ft, for both entrance and exit.
6. The further the entrance and exit lanes are from the intersection, the less interference there is with pedestrian and vehicle movement external to the lot.
7. Though the model is general enough to be applied to any combination of car sizes, we have only used combinations of compact and standard cars.
8. Problems associated with weather conditions, such as snow removal, are ignored.
9. All the dimensions are measured in feet.

TABLE 6. Eight Optimal Designs, for Different Combinations of Compact and Standard Cars

Standard car regions (1)	Compact car regions (2)	α_1 (3)	α_2 (4)	α_3 (5)	N_1 (6)	N_2 (7)	N_3 (8)	N_4 (9)	N_5 (10)	W_1 (11)	W_2 (12)	ΣN_i (13)	Ease (14)
1, 2, 3	4, 5	50	50	80	24	16	12	6	2	13.59	24.00	60	3.08
1, 2	3, 4, 5	50	45	90	24	16	14	6	2	13.59	24.00	62	2.75
1, 3	2, 4, 5	45	50	55	24	22	10	4	2	14.66	16.47	62	3.29
1	2, 3, 4, 5	45	55	85	22	22	14	6	2	13.98	24.06	66	3.15
2, 3	1, 4, 5	55	50	85	30	16	12	6	2	13.98	24.68	66	3.09
2	1, 3, 4, 5	55	50	75	32	18	14	4	2	13.98	20.53	70	2.62
3	1, 2, 4, 5	55	55	85	30	18	12	6	2	14.65	24.68	68	2.94
—	1, 2, 3, 4, 5	50	55	70	30	24	12	4	4	15.07	18.50	74	3.08

Analysis of Design

This section gives the step by step derivation of the theoretical model; it starts with an initial layout, design 1, and leads to the final initial layout, design 5, and then uses this design to derive the model, a system of nonlinear equations (Eqs. 2–11).

Preliminaries

The standard terms in parking lot design are: aisle width, denoted *AW*; curve length, *CL*; stall depth, *SD*; stall length, *SL*; stall width, *SW*, and parking angle, α. They are illustrated in Fig. 1. *SW* is a constant; *AW*, *CL*, *SD*, and *SL* are functions of α. These denote the lot parameters for standard cars. The corresponding notations for compact cars are *SSW*, *SAW*(α), *SCL*(α), *SSD*(α), and *SSL*(α). *L* and *W* are given lot dimensions. Inner turning radius, *ITR*; outer turning radius, *OTR*, and the overlap of two adjacent rows of stalls, *OV*(α) for standard cars are shown in Figs. 2(*a–b*). The corresponding parameters for compact cars are denoted *SITR*, *SOTR*, and *SOV*(α).

Basic Design Principles

After careful consideration of the problem and related material from the references, the following three obvious principles underlie the initial design. In the supporting Figs. 3–5 shaded areas represent the parking stalls, and unshaded ones are for the traffic aisles.

Principle 1

The traffic aisles should be aligned parallel to the long dimension. This minimizes the number of turning areas from one aisle to another, which results in additional parking stalls (*Parking design manual* n.d.), as shown in Fig. 3.

Principle 2

Traffic aisles should serve two rows of stalls whenever possible instead of one. This minimizes the areas used for aisles, while giving the same (*Parking design manual* n.d.) number of stalls, as shown in Fig. 4.

Principle 3

The perimeter of the parking lot should be filled with parking stalls to the maximum extent. This uses more areas for stalls and less areas for aisles (*Parking design manual* n.d.), as shown in Fig. 5.

Initial Design

Considering the third principle gives the design in Fig. 6.

From the width of the lot (100 ft), the minimum sum of stall depth and aisle width (8.5 + 12) for parallel parking ($\alpha = 0$) of a standard car, Table 1 we have

$$W_d \leq 100 - 2(8.5 + 12) = 59 \quad (1)$$

As shown in Fig. 7, a minimum distance of 25.2 ft is required between two aisles for making a U-turn, since *ITR* of a standard car is 12.6 ft (Table 1). This fact, considering the constraint of Eq. 1 and the first principle, implies that one row can be designed in the middle along the larger dimension. This limits the number of U-turns to one pair along the long dimension. Thus the layout becomes as shown in Fig. 8.

Applying the second principle of Fig. 8, the row in the middle should be divided into two adjacent rows of stalls. This completes the initial layout, shown in Fig. 9.

Design for Traffic Flow and Parking Angle

Noting Fig. 10, the counterclockwise traffic flow is selected to eliminate the cross-point shown. Considering this in design 3, assigning numbers 1–6 to its regions with parking angle variable α_i, $i = 1, \ldots, 6$, which result in different stall depths and aisle widths, and ignoring the corners at the

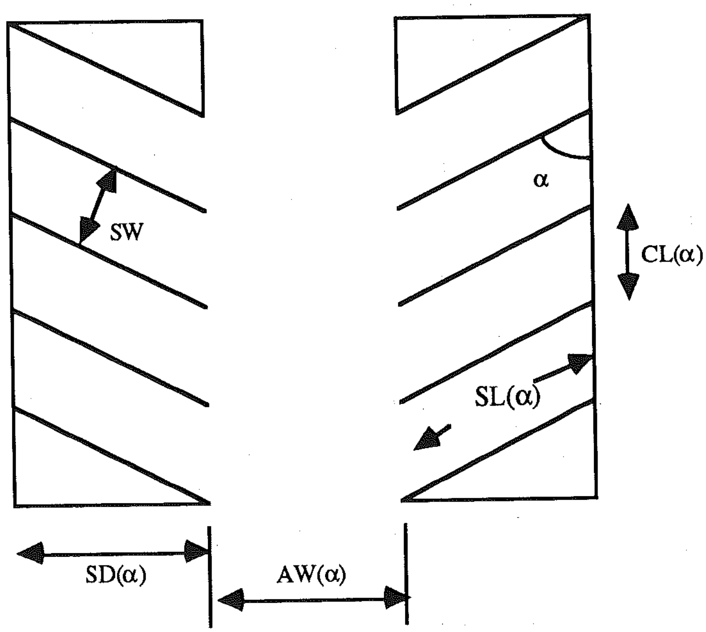


FIG. 1. Aisle and Two Rows of Stalls at Angle α

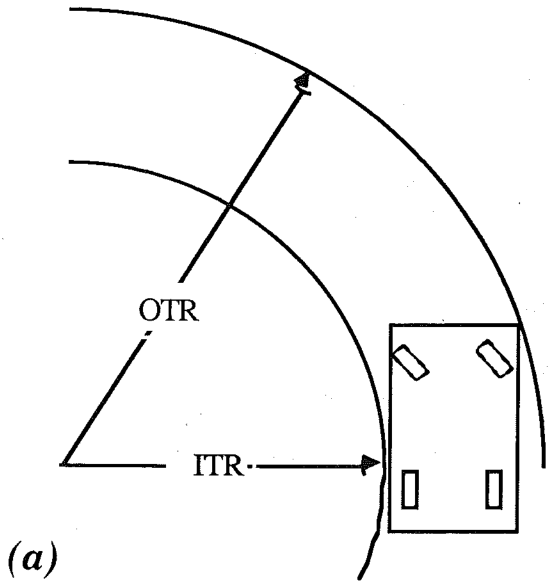


FIG. 2. (*a*) Inner and Outer Turning Radius

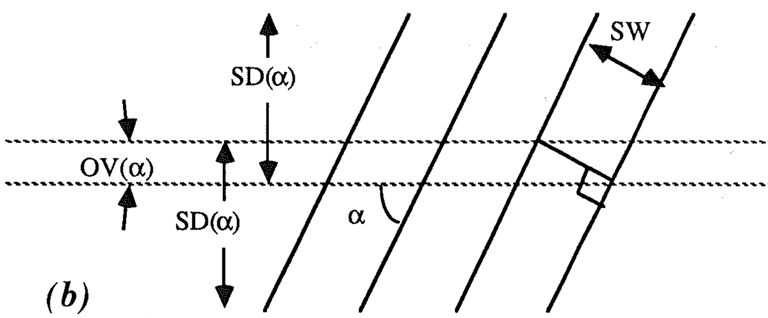


FIG. 2. (*b*) Overlap of Two Adjacent Rows

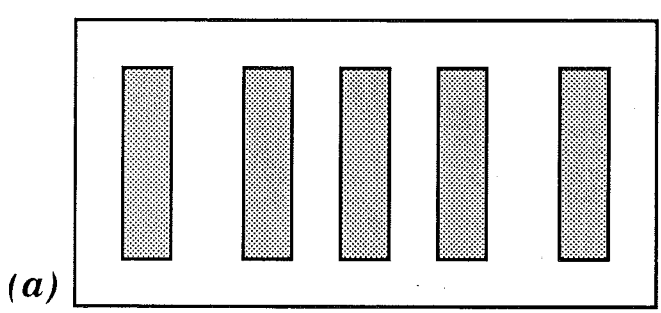
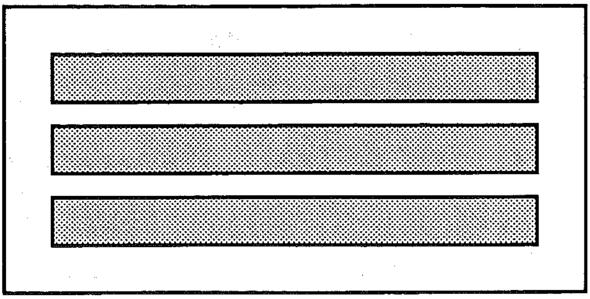
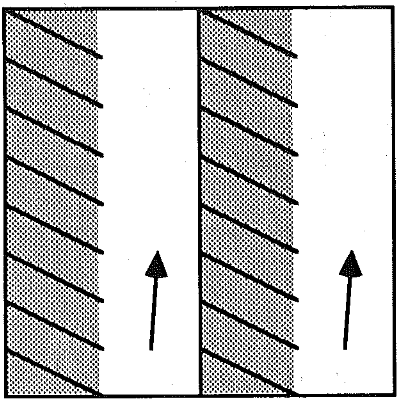


FIG. 3. (*a*) Aisles Along Short Dimension



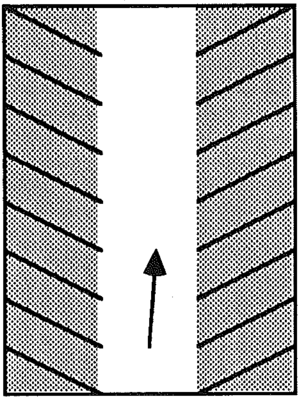
(b)

FIG. 3. (*b*) Aisles Along Long Dimension



(a)

FIG. 4. (*a*) Two Rows of Stalls and Two Aisles



(b)

FIG. 4. (*b*) Two Rows of Stalls and One Aisle

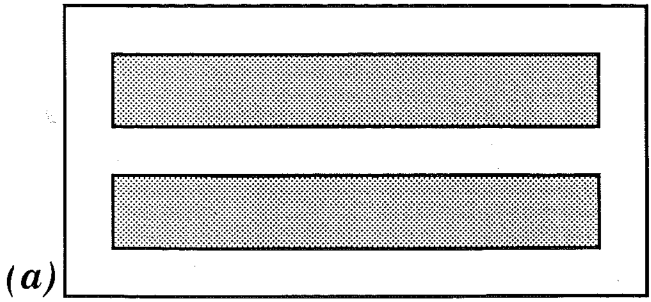


FIG. 5. (*a*) No Stalls Around Perimeter

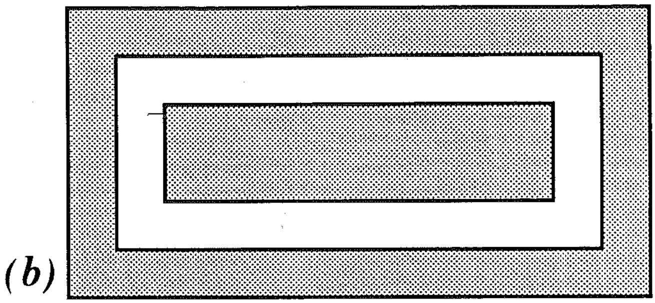


FIG. 5. (*b*) Stalls Filling Perimeter

moment, we get the design shown in Fig. 11.

We impose symmetry into the preceding design by letting $\alpha_1 = \alpha_4$; $\alpha_2 = \alpha_3$; and $\alpha_5 = \alpha_6$. This not only cuts down the computations by a big factor (see section Application) but also lessens parking confusion. Parts of the ignored corner areas of design 4 can be best used for stalls. At the same time, for better design purposes, rectangle regions 4 and 5 are incorporated near the corners (see design 5). They provide additional right angle stalls for compact cars and have the advantage of being used for straight-in park-

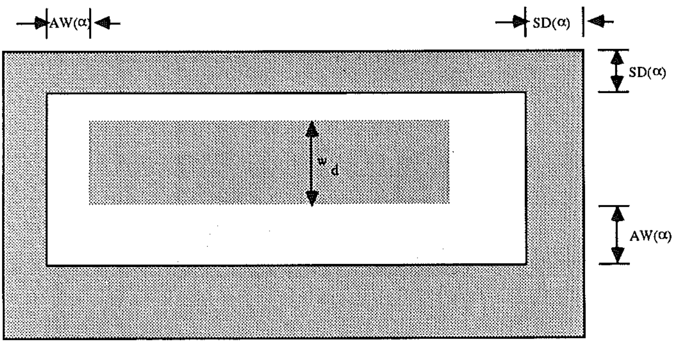


FIG. 6. Initial Layout Featuring Principle 3 (Design 1)

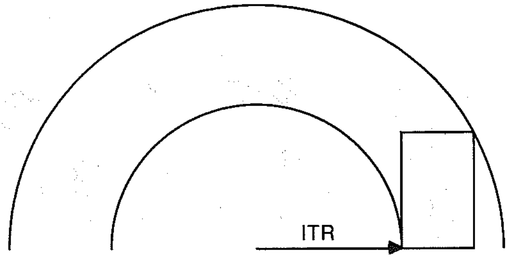


FIG. 7. Minimum Distance Required to Make U-Turn Is 2ITR

ing. One notes that portions of regions A, B, C and D will be used for the parking stalls and the remaining ones for other design purposes. Thus, design 4 after these considerations and renumbering, takes the form shown in Fig. 12.

Model

Recalling design 5, we now derive the model by first designing the aisle widths W_1 and W_2, which should be large enough for allowing easy turns to insure safety. Let

$$W_1 = \max\left(AW(\alpha_1),\ AW(\alpha_2),\ \frac{1}{2}\{W - [2SD(\alpha_1) + 2SD(\alpha_2) - OV(\alpha_2)]\}\right) \ldots \quad (2)$$

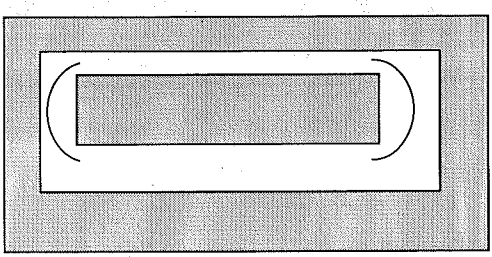


FIG. 8. Initial Layout, Featuring Principles 3, 1 and One Row in Middle (Design 2)

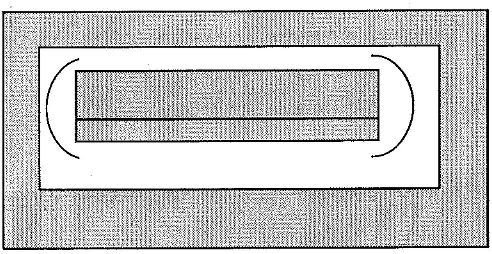


FIG. 9. Initial Layout, Featuring Principles 3, 1, One Pair of Turns, and Principle 2 (Design 3)

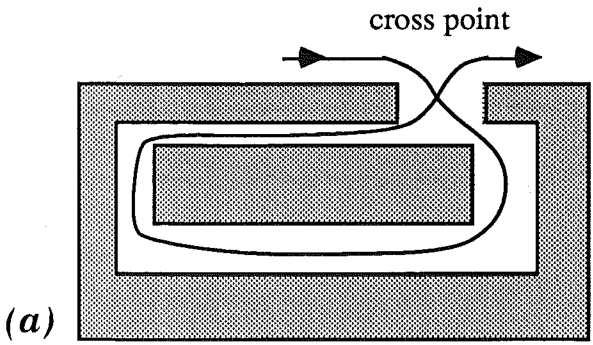


FIG. 10. ***(a)* Clockwise Flow**

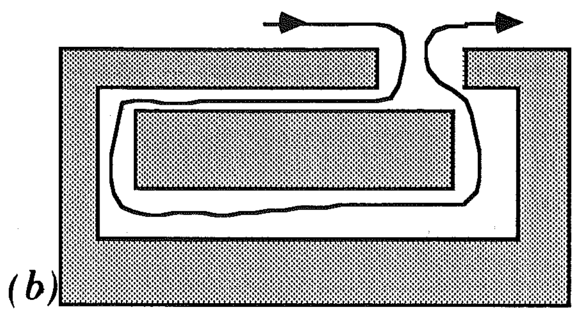


FIG. 10. ***(b)* Counter-Clockwise Flow**

where, by Fig. 2(*b*)

$$OV(\alpha_2) = \begin{cases} SW \cos(\alpha_2), & 0 < \alpha_2 \leq 90^\circ \\ 0, & \text{otherwise} \end{cases}$$

and consider the illustrations in Fig. 13. From these and simple trigonometry one can easily find

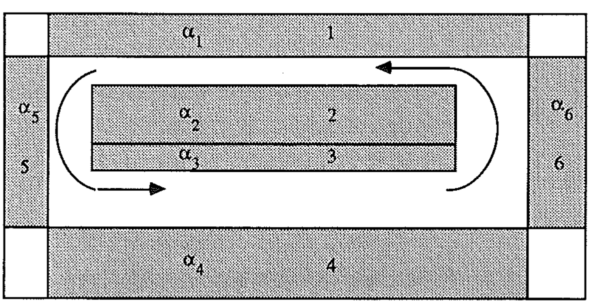


FIG. 11. Featuring Counter-Clockwise Traffic Flow and Different Parking Angles for Stalls (Design 4)

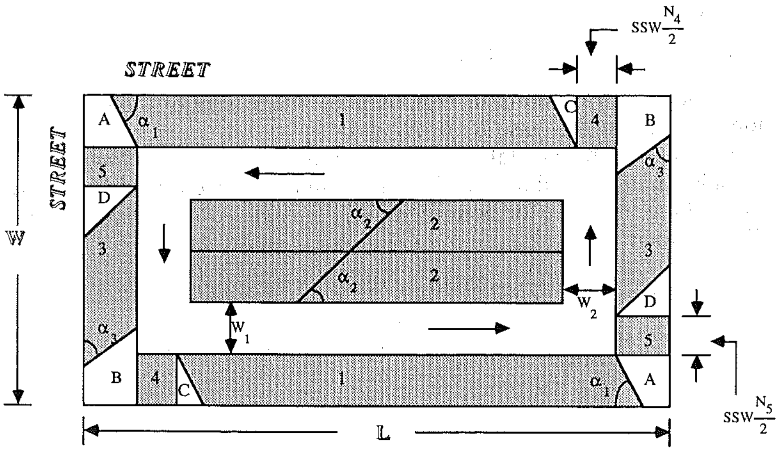


FIG. 12. Final Initial Layout, Featuring Symmetry, Stalls near Corners and Rectangles for Compact Cars and Straight in Parking (Design 5)

$$W_{2r} = \begin{cases} OTR + Clr - [ITR^2 - (OTR + Clr - W_1)^2]^{1/2}, & W_1 < OTR + Clr \\ OTR + Clr - ITR, & W_1 \geq OTR + Clr \end{cases}$$

where the clearance, Clr, is taken to be 2 ft, and thus design for

$$W_2 = \max\,[W_{2r}, AW(\alpha_3)] \qquad (3)$$

Using these we can compute the number of stalls in regions 4

$$N_4 = 2\left(\frac{W_2}{SSW}\right), \qquad \alpha_1 \geq 34° \qquad (4)$$

and regions 5

$$N_5 = 2\left(\frac{W_1}{SSW}\right), \qquad \alpha_3 \geq 34° \qquad (5)$$

noting that SSW indicates these regions can only be used for compact cars.

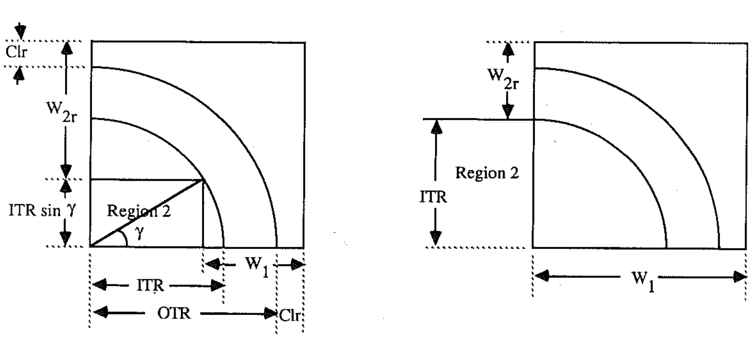


FIG. 13. Illustrations for Finding W_{2r}

If α_1 or $\alpha_3 < 34°$, then

$$SD(\alpha_i) \leq SSD(90°), \qquad i = 1 \text{ or } 3$$

implies that depth of stalls in regions 1 or 3 are smaller than the ones in regions 4 or 5. In this case N_4 or $N_5 = 0$.

To find the number of stalls in other regions of the layout shown in design 5, consider the region along L, Fig. 14(a), and calculate the number of stalls N_1, at parking angle α_1 for both regions 1. It is easy to see if $SL(\alpha_1) \cos \alpha_1 \leq SD(\alpha_3)$, then

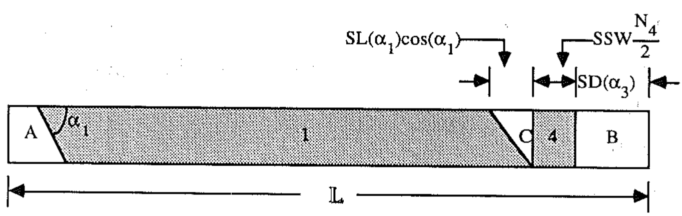
$$N_1 = 2\left[\frac{L - 2SD(\alpha_3) - SSW\dfrac{N_4}{2}}{CL(\alpha_1)}\right] \qquad (6)$$

otherwise

$$N_1 = 2\left[\frac{L - SD(\alpha_3) - SSW\dfrac{N_4}{2} - SL(\alpha_1)\cos\alpha_1}{CL(\alpha_1)}\right] \qquad (7)$$

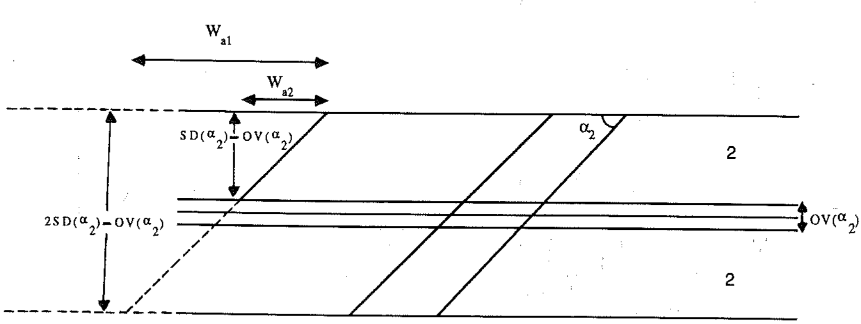
where

$$SL(\alpha_i) = \frac{SD(\alpha_i)}{\sin \alpha_i}, \qquad i = 1, 3$$



(a)

FIG. 14. (a) Region Taken from Top of the Layout in Design 5



(b)

FIG. 14. (b) Middle Part Taken from Design 5

Now consider Fig. 14(*b*), the middle part of design 5, from which for regions 2 we have

$$N_2 = 2\left(\left\{\frac{L - 2\,[SD(\alpha_3) + W_2] - W_{a1}}{CL(\alpha_2)}\right\} + \frac{W_{a2}}{CL(\alpha_2)}\right) \quad (8)$$

where

$$W_{a1} = \frac{2SD(\alpha_2) - OV(\alpha_2)}{\tan \alpha_2}, \qquad \alpha_2 \neq 0$$

and

$$W_{a2} = \frac{SD(\alpha_2) - OV(\alpha_2)}{\tan \alpha_2}, \qquad \alpha_2 \neq 0$$

or if $\alpha_2 = 0$

$$W_{a1} = W_{a2} = 0$$

Similar arguments and formulas to regions 1 hold for regions 3; i.e., if $SL(\alpha_3)\cos \alpha_3 \leq SD(\alpha_1)$, then

$$N_3 = 2\left[\frac{2SD(\alpha_2) - OV(\alpha_2) + 2W_1 - SSW\dfrac{N_5}{2}}{CL(\alpha_3)}\right] \quad (9)$$

otherwise

$$N_3 = 2\left[\frac{2SD(\alpha_2) - OV(\alpha_2) + 2W_1 - SSW\dfrac{N_5}{2} + SD(\alpha_1) - SL(\alpha_3)\cos\alpha_3}{CL(\alpha_3}\right] \quad (10)$$

One notes that Eqs. 2–10 are subject to the width constraint

$$2SD(\alpha_1) + 2W_1 + 2SD(\alpha_2) - OV(\alpha_2) \leq W = 100 \quad (11)$$

All the variables and constants for standard cars can be replaced by the correspondings for compact cars in Eqs. 2–10. In this case Eq. 4 or 5 holds for α_1 or $\alpha_3 \geq 44°$. If α_1 or $\alpha_3 < 44°$, then

$$SSD(\alpha_i) \leq SSD(90°), \qquad i = 1 \text{ or } 3$$

implies that N_4 or $N_5 = 0$.

Sources of Data

Parking lot parameters for all car sizes are given versus a few angles in *Parking Design Manual* (n.d.) and Ramsey and Sleeper (1981). Table 1 presents our selection of such data for compact cars, $SSW = 7.5$, and standard cars, $SW = 8.5$. To extend the data over all $0° \leq \alpha \leq 90°$, a natural cublic spline algorithm (Burden and Faires 1985) has been applied. The results are shown in Fig. 15. The upper curves give the parking variables $CL(\alpha)$, $SD(\alpha)$, and $AW(\alpha)$ versus $0° \leq \alpha \leq 90°$, for standard cars. The lower

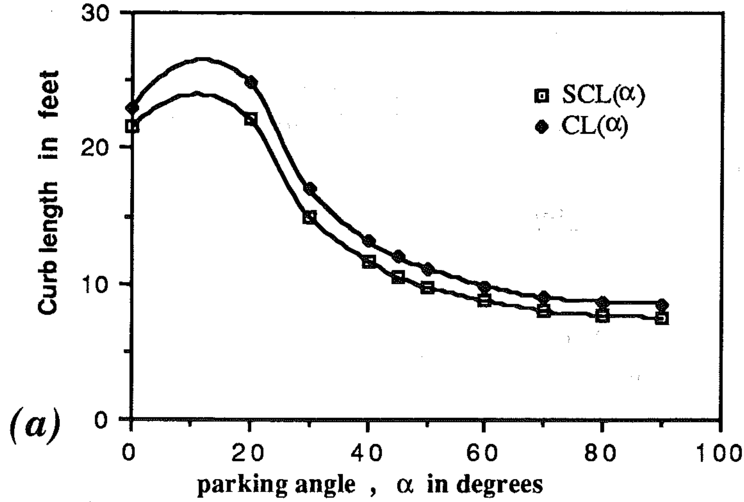


FIG. 15. (*a*) Curb Lengths for Compact and Standard Cars

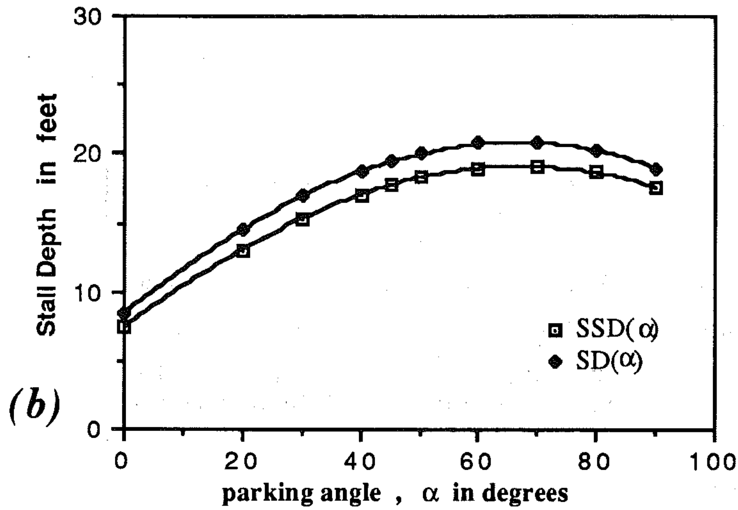


FIG. 15. (*b*) Stall Depths for Compact and Standard Cars

ones illustrate $SCL(\alpha)$, $SSD(\alpha)$, and $SAW(\alpha)$ for compact cars. The next section discusses how these generated data are used in the model.

Application of Model

The goal is to maximize ΣN_i, $i = 1, \ldots, 5$. To achieve this, we have to solve Eqs. 2–10 under the constraint of Eq. 11, for all the combinations of angles $0° \leq \alpha_i \leq 90°$, $i = 1, 2, 3$. This requires $(91)^3$ iterations. If there were no symmetry in design 5, the computations would have been much

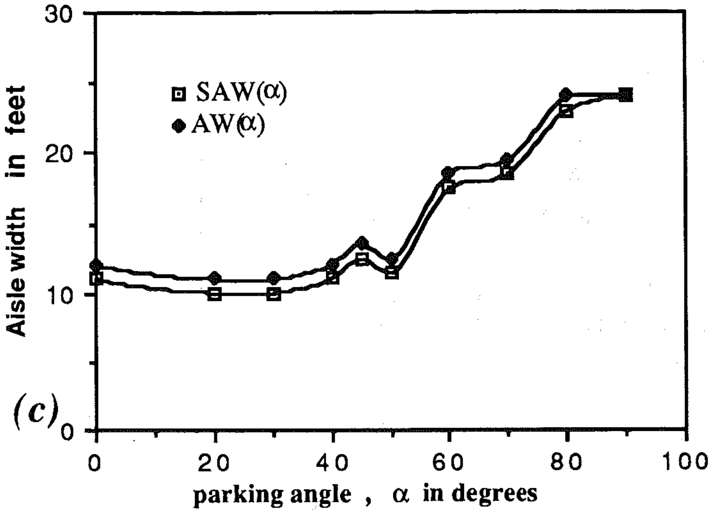


FIG. 15. **(*c*) Aisle Widths for Compact and Standard Cars**

larger, $(91)^6$. To cut down the calculations further, increments of 5° have been used. Results are generated for different combinations of compact and standard cars, in eight cases. As the sample Tables 2–5 show, each case contains the design angles, number of cars in different regions, the aisle widths, and the total number of the capacity of the lot. One notes regions 4 and 5 are to be only used for compact cars and in all the cases maximum capacities below a certain sum have been ignored. Note each N_i represents the number of stalls in two similar regions. Ease is discussed in the next section and is given by Eq. 12. It is obvious that the optimal capacity of sample Table 4 is 70 and it corresponds to one specific layout. The other sample tables give several patterns with the same maximum capacity: Table 2 gives three patterns all corresponding to 60 spaces; Table 3 gives six patterns all corresponding to 62 spaces; Table 5 gives seven patterns all corresponding to 68 spaces, and similar situations happen in other cases, which are not shown here. The next section discusses how the best pattern is selected.

Criterion for Optimal Solution, Ease

To select the optimal solution, highlighted in Tables 2, 3, and 5, we have used the last column printed under Ease. In special cases, such as Table 4, since the maximum capacity occurs once, we are not concerned with Ease. To find Ease, observe Fig. 16, which shows a typical maneuver from an aisle to a stall. From this, using basic trigonometry, the turning radius, denoted $r(\alpha)$, for a standard car is

$$r(\alpha) = \frac{d \sin\left(90 - \frac{\alpha}{2}\right)}{\sin \alpha}$$

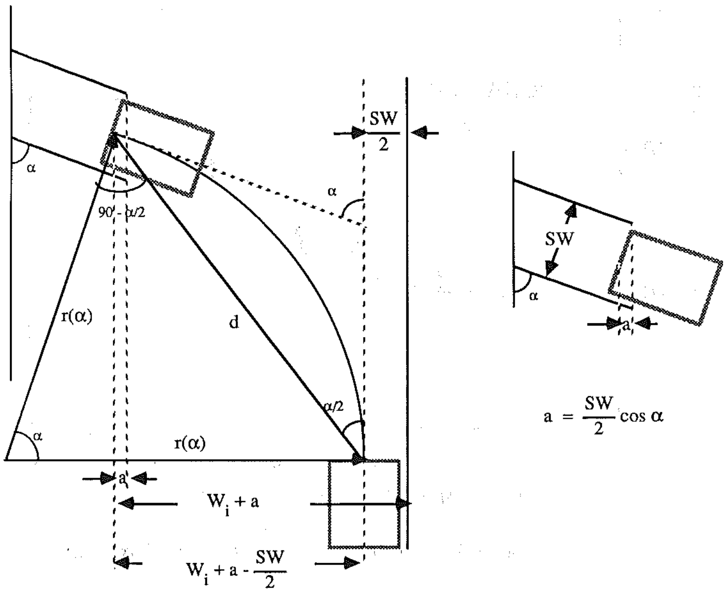


FIG. 16. A Typical Maneuver from Aisle to Stall by Standard Car

where

$$d = \frac{W_i + a - \frac{SW}{2}}{\cos\left(90 - \frac{\alpha}{2}\right)}$$

which, by substituting

$$a = \frac{SW}{2}\cos\alpha$$

$$= \frac{W_i + \frac{SW}{2}(\cos\alpha - 1)}{\cos\left(90 - \frac{\alpha}{2}\right)}, \qquad i = 1, 2$$

Upon substitution of d, the turning radius becomes

$$r(\alpha) = \frac{\left[W_i + \frac{SW}{2}(\cos\alpha - 1)\right]\tan\left(90 - \frac{\alpha}{2}\right)}{\sin\alpha}$$

Fig. 17 shows that the turning radius increases rapidly for angles below 20° for both compact and standard cars. The comfort factor for a standard car, denoted $C(\alpha)$

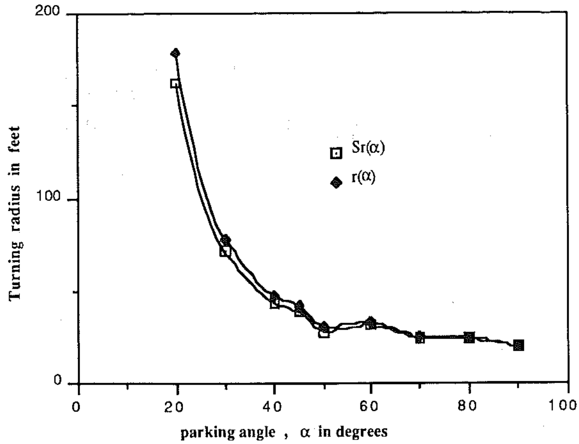


FIG. 17. Turning Radius for Compact and Standard Cars

$$C(\alpha) = \frac{r(\alpha)}{r_{\min}}$$

where

$$r_{\min} = \frac{OTR + ITR}{2}$$

represents the smallest turning radius that it can make when parking in a stall. Obviously if $C(\alpha) \leq 1$, the car can not park in one attempt. On the other hand, the bigger the turning radius, the more comfortable it is to park. If we apply $C(\alpha)$ to the regions where we have to turn to park, we get a weighted average or overall comfort

$$\bar{C} = \frac{\sum_{i=1}^{5} C(\alpha_i)N_i}{\sum_{i=1}^{5} N_i} \quad \text{.......................................} \quad (12)$$

which calculates the ease. Theoretically as can be seen from the formulas and Fig. 17, this analysis is not valid for parallel parking, $\alpha = 0$, or angles below 20°. In such cases $r(\alpha)$ gets large, which makes $C(\alpha)$ large and thus $\bar{C}$ becomes arbitrarily large. To avoid this, we have chosen a large cap for $r(\alpha)$ in the computer programs whenever the parking angles are below 20°.

FINAL DESIGN

We can now follow the previous section and use the overall comfort, Ease, given by Eq. 12, to locate the optimal layout. For example, in Table 2, out of the three layouts corresponding to a maximum 60 cars, the best is the one with the highest Ease, 3.08. Similar procedures have been used in Tables 3 and 5, and optimal patterns are selected according to the highest Ease (2.75 and 2.94, respectively). For clarity, the optimal result in each table has been highlighted, and the results for all cases are shown in Table 6.

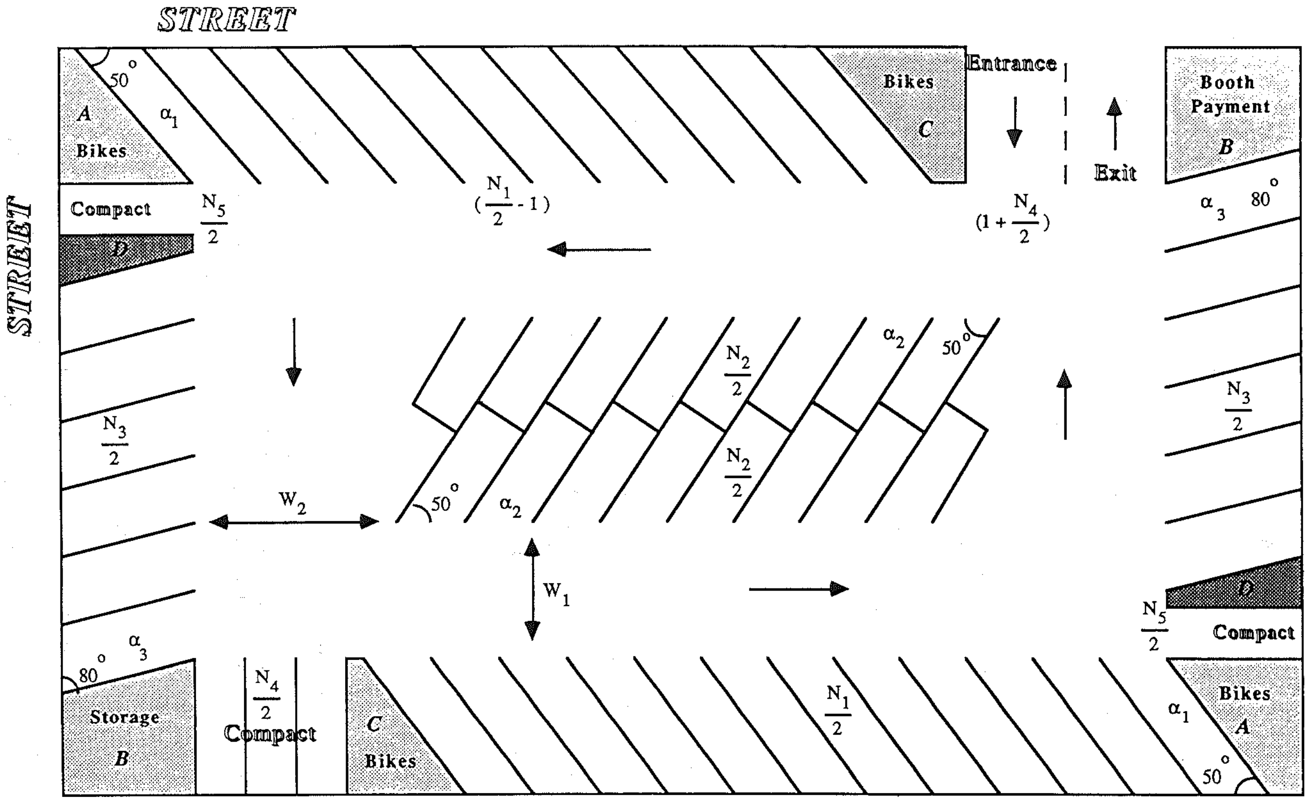


FIG. 18. The Final Layout for Sample Table 2 (Design 6)

The first and second rows consist of the optimal results of the sample Tables 2 and 3. The 6th and 7th rows correspond to sample Tables 4 and 5. We have finally applied the optimal result of sample Table 2 to design 5 to get the final layout for this case, shown in Fig. 18. From this design, we note that four spaces near the upper corner B (three from the upper region 4 and one from the upper region 1) have been taken for the entrance and exit. An automatic gate can be installed at the entrance for ticketing and a booth on the upper corner B, by the exit, for parking payment collections.

There are portions of other areas (lower B, regions A, C, and D) which can not be used for car parking spaces. The designer can make various use of these areas. For example, lower B could be for storage, and upper and lower A and C could be designed for motorcycle and bicycle spaces. Other specifications, such as landscaping, lighting, snow removal, etc., can be easily incorporated in this design as needed.

Conclusions and Interpretation

The specific result obtained is that several patterns could be designed that give the same maximum capacity (Tables 2, 3, and 5). The best pattern is then selected through Ease, a measure of comfort to park in the lot and thus a factor of safety. This is shown for the sample cases in Tables 2, 3, and 5. Optimal results of these tables and other cases are then put together in Table 6. In situations (Table 4) where there is only one maximum, we are not concerned with Ease. This is not a typical case. Finally using design 5, we show design 6 as the actual optimal layout of the sample Table 2. It is noted that four spaces are used for the gates, so in this case the maximum capacity is 56. Similar plots could easily be obtained for the rest of Table 6, using design 5.

The general features of the model are several: one is the practical layout of the initial design based on the three intuitive principles; the most important one is that the equations are general enough to handle: (1) Any car size; (2) any combination of car sizes; and (3) different angles in different regions. Another important feature is the derivation of the Ease, as a measure of comfort and a guide to locate the optimal layout from several with the same maximum capacity. In fact, the Ease measure improves on the safety factors incorporated in the derivation of the model and the conventional lot design data (Table 1), which were used as inputs for calculations.

At present the model can be used for rectangular shapes of arbitrary lengths but a fixed width. In future work, we shall try to generalize the model to handle arbitrary widths and perhaps arbitrary shapes. For the very final drawings, a package could be designed where a user could input the key parameters of Table 6 and, within seconds, obtain a computer plot indicating the optimal layouts. In this paper, we do not see the necessity of discussing the mathematical reasonings for the obvious intuitive principles.

Acknowledgment

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Appendix II. Notation

The following symbols are used in this paper:

L = length of the lot;
W = width of the lot; and
α = parking angle.

For standard cars

$AW(\alpha)$ = aisle width, a function of α;
$CL(\alpha)$ = curb length, a function of α;
ITR = inner turning radius, a constant;
OTR = outer turning radius, a constant;
$OV(\alpha)$ = overlap of two adjacent rows of stalls, a function of α;
$SD(\alpha)$ = stall depth, a function of α;
$SL(\alpha)$ = stall length, a function of α; and
SW = stall width, a constant.

For compact cars

$SAW(\alpha)$ = aisle width, a function of α;
$SCL(\alpha)$ = curb length, a function of α;
$SITR$ = inner turning radius, a constant;
$SOTR$ = outer turning radius, a constant;
$SOV(\alpha)$ = overlap of two adjacent rows of stalls, a function of α;
$SSD(\alpha)$ = stall depth, a function of α;
$SSL(\alpha)$ = stall length, a function of α; and
SSW = stall width, a constant.